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School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS: SPECIALIST

UNITS 3C AND 3D

FORMULA SHEET

2012

This document is valid for teaching and examining until 31 December 2012.

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Vectors

$|(a_1, a_2, a_3)| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ $\quad |\mathbf{a} + \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2 + a_3 b_3$

Vector equation of a line in space:

one point and the slope: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

two points A and B: $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$

Cartesian equations of a line in space: $\dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3}$

Parametric form of vector equation of a line in space:

$$x = a_1 + \lambda b_1 \ldots\ldots(1)$$
$$y = a_2 + \lambda b_2 \ldots\ldots(2)$$
$$z = a_3 + \lambda b_3 \ldots\ldots(3)$$

Vector equation of a plane in space: $\mathbf{r} \cdot \mathbf{n} = c$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

Trigonometry

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$A = \frac{1}{2}ab\sin C$$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$ Area of sector $= \frac{1}{2}r^2\theta$

Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$

$\cos^2\theta + \sin^2\theta = 1$

$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\tan(\theta \pm \varphi) = \dfrac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\quad \displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Simple Harmonic Motion: If $\dfrac{d^2x}{dt^2} = -k^2x$ then $x = A\sin(kt + \alpha)$ or $x = A\cos(kt + \beta)$

$v^2 = k^2(A^2 - x^2)$, where A = amplitude of the motion, α and β are phase angles and v = velocity and x is the displacement.

Functions

If $f(x) = y$ then $f'(x) = \frac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = \ln x$ then $f'(x) = \frac{1}{x}$

If $f(x) = \sin x$ then $f'(x) = \cos x$

If $f(x) = \cos x$ then $f'(x) = -\sin x$

If $f(x) = \tan x$ then $f'(x) = \sec^2 x = \frac{1}{\cos^2 x}$

Product rule:

If $y = f(x)\, g(x)$

then $y' = f'(x)\, g(x) + f(x)\, g'(x)$

or

If $y = uv$

then $\frac{dy}{dx} = \frac{du}{dx}\text{v} + \text{u}\frac{dv}{dx}$

Quotient rule:

If $y = \frac{f(x)}{g(x)}$

then $y' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$

or

If $y = \frac{u}{v}$

then $\frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

Incremental formula: $\delta y \simeq \frac{dy}{dx}\delta x$ or $f(x+h) - f(x) \simeq f'(x)h$

Chain rule:

If $y = f(g(x))$

then $y' = f'(g(x))\, g'(x)$

or

If $y = f(u)$ and $u = g(x)$

then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Powers: $\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Logarithms: $\int \frac{1}{x} dx = \ln|x| + c$

Trigonometric: $\int \sin x\, dx = -\cos x + \text{c}$

$\int \cos x\, dx = \sin x + \text{c}$

$\int \frac{1}{\cos^2 x}\, dx = \tan x + \text{c}$

Fundamental Theorem of Calculus:

$\frac{\text{d}}{\text{d}x}\int_a^x f(t)\,dt = f(x)$ and $\int_a^b f'(x)\,dx = f(b) - f(a)$

Functions

Quadratic function:

If $y = ax^2 + bx + c$ and $y = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $x \in C$

Piecewise-defined functions:

Absolute value function: $|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Sign function: $\text{sgn}\,(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

Greatest integer function:
$\text{int}\,(x)$ = greatest integer $\leq x$ for all x

Matrices

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = \det A = ad - bc$

$A^{-1} = \dfrac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Dilation = $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Shear = $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Rotation = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection = $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

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Complex numbers

For $z = a + ib$, where $i^2 = -1$

Argument: $\text{Arg}\, z = \theta$, where $\tan\theta = \dfrac{b}{a}$ and $-\pi < \theta \leq \pi$

Modulus: $\text{mod}\, z = |z| = |a + ib| = \sqrt{a^2 + b^2} = r$

Product: $|z_1 z_2| = |z_1|\,|z_2|$

$\arg z_1 z_2 = \arg z_1 + \arg z_2$

Quotient: $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

$\arg \dfrac{z_1}{z_2} = \arg z_1 - \arg z_2$

Polar form:
For $z = r\,\text{cis}\,\theta$, where $r = |z|$ and $\theta = \arg z$:

$\text{cis}(\theta + \varphi) = \text{cis}\theta\,\text{cis}\varphi$

$\text{cis}\theta = \cos\theta + i\sin\theta$

$\text{cis}(-\theta) = \dfrac{1}{\text{cis}\theta}$

$\text{cis}(0) = 1$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta + \varphi)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta - \varphi)$

Exponential form:
$z = re^{i\theta}$, where $r = |z|$ and $\theta = \arg z$

For complex conjugates:

$z = a + bi$

$\bar{z} = a - bi$

$z = r\,\text{cis}\,\theta$

$\bar{z} = r\,\text{cis}\,(-\theta)$

$z = re^{i\theta}$

$\bar{z} = re^{-i\theta}$

$z\bar{z} = |z|^2$

$z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Exponentials and logarithms

For a, $b > 0$ and m, n real:

$a^m a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

For m an integer and n a positive integer:

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

For a, b, y, m and n positive real and k real:

$1 = a^0 \Leftrightarrow \log_a 1 = 0$

$y = a^x \Leftrightarrow \log_a y = x$

$\log_a(mn) = \log_a(m) + \log_a(n)$

$a = a^1 \Leftrightarrow \log_a a = 1$

$\log_a(m) = \dfrac{\log_b(m)}{\log_b(a)}$ (change of base)

$\log_a(m^k) = k \log_a(m)$

If $\dfrac{dP}{dt} = kP$, then $P = P_0 e^{kt}$

Mathematical reasoning

De Moivre's theorem:

$(\text{cis}\ \theta)^n = (\cos\theta + i\sin\theta)^n$

$(\text{cis}\ \theta)^n = \cos n\theta + i\sin n\theta$

$z^n = |z|^n \text{cis}\ (n\theta)$

$z^{\frac{1}{q}} = |z|^{\frac{1}{q}}\left(\cos\left(\dfrac{\theta + 2\pi k}{q}\right) + i\sin\left(\dfrac{\theta + 2\pi k}{q}\right)\right)$ for k an integer.

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height

Prism: $V = Ah$, where V is the volume, A is the area of the base and h is the perpendicular height

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.